MATH 1A: GIVEN $y = f(x)$

FIND $y' = f'(x)$ or $\frac{dy}{dx} = f'(x)$

MATH 1B: GIVEN $\frac{dy}{dx} = g(x)$

FIND $y = \int g(x)\,dx$

BUT ~~WHAT~~ IF $\frac{dy}{dx} = g(y)$ or $\frac{dy}{dx} = g(x,y)$ ?

$$\frac{dy}{dx} = \frac{x-5}{y^2}$$

eg.

$$\frac{d^2y}{dx^2} = y(x^2 + \cos y)$$

OR $xy'' + y' + xy = 0$

# DIFFERENTIAL EQUATIONS

INDEPENDENT VARIABLES

DEPENDENT VARIABLES


EQUATION INVOLVING
DERIVATIVES OF DEPENDENT VARIABLES
WITH RESPECT TO INDEPENDENT VARIABLES
POSSIBLY INVOLVING THE DEPENDENT +
INDEPENDENT VARIABLES

A = AMOUNT OF RADIOACTIVE MATERIAL AT TIME $t$

$$\frac{dA}{dt} = -kA \quad (k>0)$$

INDEPENDENT VARIABLE I.V. IS $t$
DEPENDENT D.V. IS $A$

$$\frac{dA}{dt} = -kA \longrightarrow \frac{1}{A}\,dA = -k\,dt$$

$$\int \frac{1}{A}\,dA = \int -k\,dt$$

$$\ln|A| + C_1 = -kt + C_2$$

$$\ln|A| = -kt + C$$

$$|A| = e^{-kt+C}$$

$$|A| = e^{-kt}e^{C}$$

$$|A| = A_0 e^{-kt}$$

$$A = \pm A_0 e^{-kt}$$

$$A = A_0 e^{-kt}$$

ORDER OF D.E. = HIGHEST ORDER DERIVATIVE INVOLVED IN EQUATION

$$x\frac{dy}{dx} - y^2 = \cos x \qquad \text{ORDER} = 1$$

$$\sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2} = y \qquad \text{ORDER} = 2$$

$$yy''' - xy^{(4)} = 0 \qquad \text{ORDER} = 4$$

$$\frac{\partial^3 z}{\partial y^3} + \frac{\partial^2 z}{\partial x^2} = x^2 y \qquad \text{ORDER} = 3$$

(ODE) → ORDINARY DIFFERENTIAL EQUATION → 1 DEPENDENT VARIABLE IS FUNCTION OF INDEPENDENT VARIABLE

PARTIAL DIFFERENTIAL EQUATION → >1 " "

(PDE) ↑

n-ORDER ODES CAN BE WRITTEN IN FORM

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}\right) = 0$$

OR $$\frac{d^ny}{dx^n} = G\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots \frac{d^{n-1}y}{dx^{n-1}}\right)$$

LINEAR ODE

$$Ax + By + Cz = D$$

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + a_{n-2}(x)\frac{d^{n-2} y}{dx^{n-2}} + \cdots$$

$$+ a_1(x)\frac{dy}{dx} + a_0(x)y = F(x)$$

$$xy'' + (\cos x)y' - 3y = e^x \quad \text{LINEAR}$$

$$(y')^2 = x \quad \text{NOT LINEAR}$$